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Elastic-plastic transition in a transversely isotropic thin rotating disc having variable density with edge loading

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ABSTRACT
The paper, investigates the influence of density on the elastic-plastic stresses in a transversely isotropic thin rotating disc with edge loading using Seth's transition theory. The effects of angular speed have been discussed for initial yielding and fully- plastic state. A thin rotating disc made of isotropic material (Brass) whose density increases rapidly requires higher percentage increase in angular speed to become fully- plastic as compared to rotating disc having constant density or whose density decreases rapidly and made of transversely isotropic material. Rotating disc having variable density and made of isotropic material have a tendency to fracture at bore i.e., it is where the largest tensile stress occurs as compared to rotating disc made of transversely isotropic material. The tendency of fracture at the bore increases with the increases in edge loading.

KEY WORDS: Stresses, strain, transversely isotropic material, rotating disc, density, and edge load

INTRODUCTION

Rotating disc forms an essential part of the design of rotating machinery *viz.* rotors, turbines, compressors and flywheel etc. The use of rotating disc in machinery and structural applications has generated considerable interest in many problems in the domain of solid mechanics. Solutions for thin isotropic discs can be found in most of the standard elasticity and plasticity textbooks [1-4]. Reddy and Srinath [5] investigated the influence of material density on the stress and displacement of a rotating polar orthotropic circular disc. It has been shown that the existence of a density gradient in a rotating disc influences the stresses and displacements significantly. Chang [6] developed closed form solutions for a rotating orthotropic circular disc with variable density. Guven [7] found the elastic–plastic stresses in rotating annular disc of variable thickness and variable density under the assumption of Tresca's yield condition, its associated flow rule and linear strain hardening. To obtain the stress distribution, Guven matched the elastic-plastic stresses at the same radius r = z of the disc. Perfect elasticity and ideal plasticity are two extreme properties of the material and the use of an ad-hoc rule like yield condition amounts to divide the two extreme properties by a sharp line which is not physically possible. When a material passes from one state to another qualitatively different state, transition takes place. Since this transition is non-linear in character and difficult to investigate, workers have taken certain ad-hoc assumptions like yield condition, incompressibility condition and a strain law, which may or may not valid for the problem. Seth's transition theory [8] does not require these assumptions and thus poses and solves a more general problem, from which cases pertaining to the above assumptions can be worked out. This theory utilizes the concept of generalized strain measure and asymptotic solution at the critical points or turning points of the differential equations defining the deformed field and has been successfully applied to lager number of the problems [9-14].

Here the influence of density on the elastic-plastic stresses in a transversely isotropic thin rotating disc is investigated with edge loading using Seth's transition theory. The density of the disc is assumed to vary along the radius in the form

$$\rho = \rho_o \left(\frac{r}{b}\right)^{-m}, \qquad (1)$$

Where, ρ_o is the density at r = ' b' and m is the density parameter. Results obtained have been discussed numerically and depicted graphically.

Governing Equations

Consider a thin circular disc of variable density with a central bore of radius ‘a’ and external radius ‘ b’ rotating with angular velocity ω of gradually increasing magnitude about an axis perpendicular to its plane and passing through the centre. The thickness of the disc is assumed to be constant and is taken sufficiently small so that the disc is effectively in a state of plane stress i.e. the axial stress T_{zz} is zero.

In cylindrical polar co-ordinates the displacements are given by,

$$u = r(1-\beta);\ v = 0 \text{ and } w = d\ .\ z \qquad (2)$$

where β is a function of $r = \sqrt{x^2 + y^2}$ only and d is a constant.

The generalized components of strain are,

$$e_{rr} = \frac{1}{n}\left[1 - \left(\beta + r\beta'\right)^n\right],$$

$$e_{\theta\theta} = \frac{1}{n}\left[1 - \beta^n\right], \qquad (3)$$

$$e_{zz} = \frac{1}{n}\left[1 - (1-d)^n\right], \qquad (4)$$

$e_{r\theta} = e_{\theta z} = e_{zr} = 0$, where n is the measure and $\beta' = \dfrac{d\beta}{dr}$.

Stress-strain relations for this problem becomes,

$$T_{rr} = C_{11}e_{rr} + (C_{11} - 2C_{66})e_{\theta\theta} + C_{13}\,e_{zz},$$

$$T_{\theta\theta} = (C_{11} - 2C_{66})e_{rr} + C_{11}\,e_{\theta\theta} + C_{13}\,e_{zz},$$

$$T_{zr} = T_{\theta z} = T_{r\theta} = T_{zz} = 0.$$

Substituting equations (3) in (4), the non-zero stress components are,

$$T_{rr} = \frac{A}{n}\left[2 - \beta^n\left\{1 + (1+P)^n\right\}\right] - 2\frac{C_{66}}{n}\left[1 - \beta^n\right],$$

$$T_{\theta\theta} = \frac{A}{n}\left[2 - \beta^n\left\{1 + (1+P)^n\right\}\right] - 2\frac{C_{66}}{n}\left[1 - \beta^n(1+P)^n\right], \qquad (5)$$

$$T_{zr} = T_{\theta z} = T_{r\theta} = T_{zz} = 0,$$

where $A = C_{11} - \dfrac{C_{13}^2}{C_{33}}$ and $r\beta' = \beta P$.

The equations of equilibrium are all satisfied except

$$\frac{d}{dr}\left(rT_{rr}\right) - T_{\theta\theta} + \rho r^2\omega^2 = 0. \qquad (6)$$

Using equation (5) in (6), we get a non-linear differential equation in β as,

$$P\beta^{n+1}(1+P)^{n-1}\frac{dP}{d\beta} = \left[\begin{array}{l} -P\beta^n\left[1+(1+P)^n\right] + \dfrac{2C_{66}}{nA}\beta^n\left[1-(1+P)^n\right] \\ + \dfrac{2C_{66}}{A}P\beta^n + \dfrac{\rho r^2\omega^2}{A}\end{array}\right], \qquad (7)$$

The transitional points of β in equation (7) are $P \to -1$ and $P \to \pm\infty$.

The boundary conditions are

$T_{rr} = 0$ at r = ‘a’ and $T_{rr} = T_0$ at r = ‘ b’. (8)

Solution through the principal stress

It has been shown [11, 15-17] that the asymptotic solution through the principal stress leads from elastic state to plastic state at the transition point $P \to \pm\infty$. For finding the plastic stress at the transition point $P \to \pm\infty$, We define the transition function R_2 as,

$$R_2 \equiv T_{\theta\theta} = \frac{A}{n}\left[2 - \beta^n\left\{1 + (1+P)^n\right\}\right] - 2\frac{C_{66}}{n}\left[1 - \beta^n(1+P)^n\right]. \qquad (9)$$

Taking the logarithmic differentiation and substituting the value of $\dfrac{dP}{d\beta}$ from equation (7) in equation (9), we get,

$$\frac{d}{dr}(\log R_2) = \frac{-A}{rR_2}\left[\begin{array}{l} -P\beta^n\left\{1+(1+P)^n\right\} + \dfrac{2C_{66}}{nA}\beta^n - \dfrac{2C_{66}}{nA}\beta^n(1+P)^n + \dfrac{4C_{66}}{A}P\beta^n \\ + \dfrac{\rho r^2\omega^2}{A} - \dfrac{4C_{66}^2}{nA^2}\beta^n\left[1-(1+P)^n\right] - \dfrac{4C_{66}^2}{A^2}P\beta^n - \dfrac{2C_{66}}{A^2}\rho r^2\omega^2\end{array}\right] \qquad 10)$$

(4)

Taking the asymptotic value as $P \to \pm\infty$ in equation (10), we get,

$$\frac{d}{dr}\left(\log R_2\right) = \frac{-C_2}{r},$$

where $C_2 = \dfrac{2C_{66}}{A}$ and $A = C_{11} - \dfrac{C_{13}^2}{C_{33}}$. (11)

where $C_2 = \dfrac{2C_{66}}{A}$ and $A = C_{11} - \dfrac{C_{13}^2}{C_{33}}$.

Integration of equation (11) gives,

$R_2 = A_1 r^{-C_2}$, where A_1 is constant of integration. (12)

From equation (9) and equation (11), we have,

$$T_{\theta\theta} = A_1 r^{-C_2} . \qquad (13)$$

Substituting equation (13) in equation (6) and integrating, we get,

$$rT_{rr} = \frac{A_1 r^{1-C_2}}{1-C_2} - \omega^2 \int \rho r^2 dr + A_2 . \qquad (14)$$

where A_2 is another integrating constant.

Using boundary conditions (8) in equation (14), we get,

$$A_1 = (1-C_2)\left[\frac{bT_0 + \omega^2 \int_a^b \rho r^2 dr}{b^{1-C_2} - a^{1-C_2}}\right]$$

and

$$A_2 = -\left[\frac{bT_0 + \omega^2 \int_a^b \rho r^2 dr}{b^{1-C_2} - a^{1-C_2}}\right] a^{1-C_2} + \omega^2 \left[\int \rho r^2 dr\right]_{at\ r=a} .$$

(15)

Substituting the values of A_1 and A_2 from equation (15) in equations (13) and (14), we get transitional stresses as,

$$T_{rr} = \frac{1}{r}\left[\frac{r^{1-C_2} - a^{1-C_2}}{b^{1-C_2} - a^{1-C_2}}\right]\left[bT_0 + \frac{\rho_0 \omega^2 b^m}{3-m}\left(b^{3-m} - a^{3-m}\right)\right] - \frac{\rho_0 \omega^2 b^m}{r(3-m)}\left[r^{3-m} - a^{3-m}\right]$$

, (16)

$$T_{\theta\theta} = \frac{(1-C_2) r^{-C_2}}{b^{1-C_2} - a^{1-C_2}}\left[bT_0 + \frac{\rho_0 \omega^2 b^m}{3-.m}\left(b^{3-m} - a^{3-m}\right)\right].$$

for $m \neq 3$ (17)

Initial Yielding

It is seen from equation (17) that $T_{\theta\theta}$ is maximum at the internal surface (r = a). Therefore, yielding in the disc will take place at the internal surface and equation (17) becomes, for $m \neq 3$

$$\left|T_{\theta\theta}\right|_{r=a} = \left|\frac{(1-C_2) a^{-C_2}}{b^{1-C_2} - a^{1-C_2}}\left[bT_0 + \frac{\rho_0 \omega^2 b^m}{3-m}\left(b^{3-m} - a^{3-m}\right)\right]\right| \equiv Y$$

(Say). (18)

Angular speed Ω_i^2 for initial yielding (at internal surface) is given by,

$$\Omega_i^2 = \frac{\rho_0 \omega_i^2 b^2}{Y} = \frac{(3-m) b^{2-m}}{b^{3-m} - a^{3-m}}\left[\frac{b^{1-C_2} - a^{1-C_2}}{(1-C_2) a^{-C_2}} - b\sigma_0\right]$$

. (19)

We introduce the following non-dimensional components as,

$$R = \frac{r}{b},\ R_0 = \frac{a}{b},\ \Omega_i^2 = \frac{\rho_o \omega_i^2 b^2}{Y},\quad \sigma_r = \frac{T_{rr}}{Y},$$

$$\sigma_\theta = \frac{T_{\theta\theta}}{Y},\quad \Omega_f{}^2 = \frac{\rho_o \omega_f{}^2 b^2}{Y} \quad and \quad \sigma_0 = \frac{T_0}{Y}.$$

Equation (16) and (17) in non-dimensional components become,

$$\sigma_r = \frac{1}{R}\left[\frac{R^{1-C_2} - R_0^{1-C_2}}{1-R_0^{1-C_2}}\right]\left[\sigma_0 + \frac{\Omega_i^2}{3-m}\left(1-R_0^{3-m}\right)\right] - \frac{\Omega_i^2}{R(3-m)}\left[R^{3-m} - R_0^{3-m}\right]$$

, (20)

$$\sigma_\theta = \frac{(1-C_2) R^{-C_2}}{1-R_0^{1-C_2}}\left[\sigma_0 + \frac{\Omega_i^2}{3-m}\left(1-R_0^{3-m}\right)\right], \quad (21)$$

where

$$\Omega_i^2 = \frac{\rho_0 \omega_i^2 b^2}{Y} = \frac{(3-m)}{1-R_o^{3-m}}\left[\frac{R_o^{C_2}\left(1-R_o^{1-C_2}\right)}{(1-C_2)} - \sigma_0\right]$$

and $m \neq 3$.

For m=3, the stresses (20) and (21) become,

$$\sigma_r = \frac{1}{R}\left[\frac{R^{1-C_2} - R_0^{1-C_2}}{1-R_0^{1-C_2}}\right]\left[\sigma_0 - \Omega_i^2 \log R_o\right] - \frac{\Omega_i^2}{R}\log\left[\frac{R}{R_o}\right]$$

, (22)

$$\sigma_\theta = \frac{(1-C_2) R^{-C_2}}{1-R_0^{1-C_2}}\left[\sigma_0 - \Omega_i^2 \log R_o\right], \quad (23)$$

where

$$\Omega_i^2 = \frac{\rho_0 \omega_i^2 b^2}{Y} = \frac{-1}{\log R_o}\left[\frac{R_o^{C_2}\left(1-R_o^{1-C_2}\right)}{(1-C_2)} - \sigma_0\right].$$

Equations (20)-(23) gives the elastic-plastic transitional stresses for a thin rotating disc having variable density with edge loading.

Fully Plastic State

From equation (17), the angular velocity ($\omega_f > \omega_i$) required for the disc to become fully plastic $(C_2 \to 0)$ at r = b is given by,

$$\Omega_f^2 = \frac{\rho_0 \omega_f^2 b^2}{Y} = \frac{(3-m)}{1-R_o^{3-m}}\left[(1-R_o) - \sigma_o\right].$$

For $m \neq 3$ (24)

Stresses (20) and (21) for fully plastic state $(C_2 \to 0)$ become,

$$\sigma_r = \frac{1}{R}\left[\frac{R-R_0}{1-R_0}\right]\left[\sigma_o + \frac{\Omega_f^2}{3-m}\left(1-R_0^{3-m}\right)\right] - \frac{\Omega_f^2}{R(3-m)}\left[R^{3-m} - R_0^{3-m}\right]$$

,

$$\sigma_\theta = \frac{1}{1-R_0}\left[\sigma_o + \frac{\Omega_f^2}{3-m}\left(1-R_0^{3-m}\right)\right]. \qquad \text{For}$$

$m \neq 3$ (25)

For m=3, the Stresses (25) become,

$$\sigma_r = \frac{1}{R}\left[\frac{R-R_0}{1-R_0}\right]\left[\sigma_o - \Omega_f^2 \log R_0\right] - \Omega_f^2 \log\left[\frac{R}{R_o}\right],$$

$$\sigma_\theta = \frac{1}{1-R_0}\left[\sigma_o - \Omega_f^2 \log R_o\right], \qquad (26)$$

where

$$\Omega_f^2 = \frac{\rho_0 \omega_f^2 b^2}{Y} = \frac{-1}{\log R_o}\left[(1-R_o) - \sigma_o\right].$$

For a disc having Constant Density (m=0)

The stresses given by equation (25) for a disc having constant density (m=0) are given by,

$$\sigma_r = \frac{1}{R}\left[\frac{R-R_0}{1-R_0}\right]\left[\sigma_o + \frac{\Omega_f^2}{3}\left(1-R_0^3\right)\right] - \frac{\Omega_f^2}{3R}\left[R^3 - R_0^3\right]$$

,

$$\sigma_\theta = \frac{1}{1-R_0}\left[\sigma_o + \frac{\Omega_f^2}{3}\left(1-R_0^3\right)\right]. \qquad (27)$$

Isotropic Case

For isotropic materials, the material constants reduce to two only , i.e.,

$C_{11} = C_{22} = C_{33}$,

$C_{12} = C_{21} = C_{13} = C_{31} = C_{23} = C_{32} = (C_{11} - 2C_{66})$.

In terms of Lame's constant λ and μ, these can be written as,

$C_{12} = \lambda$, $C_{66} = \frac{1}{2}(C_{11} - C_{12}) \equiv \mu$ and $C_{11} = \lambda + 2\mu$. (28)

Elastic-plastic transitional stresses are obtained by using equation (28) in equation (16) and (17), as,

$$\sigma_r = \frac{1}{R}\left[\frac{R^{\frac{1-C}{2-C}} - R_0^{\frac{1-C}{2-C}}}{1-R_0^{\frac{1-C}{2-C}}}\right]\left[\sigma_0 + \frac{\Omega_i^2}{3-m}\left(1-R_0^{3-m}\right)\right] - \frac{\Omega_i^2}{R(3-m)}\left[R^{3-m} - R_0^{3-m}\right]$$

$$\sigma_\theta = \left[\frac{1-C}{2-C}\right]\frac{R^{\frac{-1}{2-C}}}{1-R_0^{\frac{1-C}{2-C}}}\left[\sigma_0 + \frac{\Omega_i^2}{3-m}\left(1-R_0^{3-m}\right)\right], \quad (29)$$

where

$$\Omega_i^2 = \frac{\rho_0 \omega_i^2 b^2}{Y} = \frac{(3-m)}{1-R_o^{3-m}}\left[\left(\frac{2-C}{1-C}\right)\left\{R_o^{\frac{1}{2-C}} - R_o\right\} - \sigma_o\right]$$

, $m \neq 3$ and $C = \dfrac{2\mu}{\lambda + 2\mu} = \dfrac{1-2\sigma}{1-\sigma}$.

For m=3, Stresses (29) becomes,

$$\sigma_r = \frac{1}{R}\left[\frac{R^{\frac{1-C}{2-C}} - R_0^{\frac{1-C}{2-C}}}{1-R_0^{\frac{1-C}{2-C}}}\right]\left[\sigma_0 - \Omega_i^2 \log R_o\right] - \Omega_i^2 \log\left[\frac{R}{R_o}\right],$$

$$\sigma_\theta = \left[\frac{1-C}{2-C}\right]\frac{R^{\frac{-1}{2-C}}}{1-R_0^{\frac{1-C}{2-C}}}\left[\sigma_0 - \Omega_i^2 \log R_o\right], \quad (30)$$

where

$$\Omega_i^2 = \frac{\rho_0 \omega_i^2 b^2}{Y} = \frac{-1}{\log R_o}\left[\left(\frac{2-C}{1-C}\right)\left\{R_o^{\frac{1}{2-C}} - R_o\right\} - \sigma_o\right].$$

Fully -Plastic State (Isotropic case)

From equation (30), the angular velocity ($\omega_f > \omega_i$) required for the disc to become fully plastic $(C \to 0)$ at r = b is given by,

$$\Omega_f^2 = \frac{\rho_0 \omega_f^2 b^2}{Y} = \frac{(3-m)}{1-R_o^{3-m}}\left[2(1-\sqrt{R_o}) - \sigma_o\right], \quad (31)$$

Stresses for fully plastic case ($C \to 0$) are obtained as,

$$\sigma_r = \frac{1}{R}\left[\frac{\sqrt{R}-\sqrt{R_0}}{1-\sqrt{R_0}}\right]\left[\sigma_o + \frac{\Omega_f^2}{3-m}\left(1-R_0^{3-m}\right)\right] - \frac{\Omega_f^2}{R(3-m)}\left[R^{3-m} - R_0^{3-m}\right]$$

,

$$\sigma_\theta = \frac{1}{2\sqrt{R}\left(1-\sqrt{R_0}\right)}\left[\sigma_o + \frac{\Omega_f^2}{3-m}\left(1-R_0^{3-m}\right)\right].$$

For $m \neq 3$ (32)

For m=3, Stresses (32) becomes

$$\sigma_r = \frac{1}{R}\left[\frac{\sqrt{R}-\sqrt{R_0}}{1-\sqrt{R_0}}\right]\left[\sigma_o - \Omega_f^2 \log R_o\right] - \Omega_f^2 \log\left[\frac{R}{R_o}\right]$$

$$, \sigma_\theta = \frac{1}{2\sqrt{R}\left(1-\sqrt{R_0}\right)}\left[\sigma_o - \Omega_f^2 \log R_o\right], \qquad (33)$$

where $\Omega_f^2 = \dfrac{\rho_0 \omega_f^2 b^2}{Y} = \dfrac{-1}{\log R_o}\left[2(1-\sqrt{R_o}) - \sigma_o\right]$.

Equations (32) are same as obtained by Gupta, Sharma and Pathak [18].

For a disc having Constant Density (isotropic case, m=0)

The stresses (33) for a disc having constant density are obtained as

$$\sigma_r = \frac{1}{R}\left[\frac{\sqrt{R}-\sqrt{R_0}}{1-\sqrt{R_0}}\right]\left[\sigma_o + \frac{\Omega_f^2}{3}\left(1-R_0^3\right)\right] - \frac{\Omega_f^2}{3R}\left[R^3 - R_0^3\right]$$

...34

$$\sigma_\theta = \frac{1}{2R\left(1-\sqrt{R_0}\right)}\left[\sigma_o + \frac{\Omega_f^2}{3}\left(1-R_0^3\right)\right]. \quad (34)$$

These equations are same as obtained by Gupta and Shukla [12].

Numerical Illustration and Discussion (Flat Disc)

As a numerical example, elastic constants C_{ij} have been given in Table 1 for transversely isotropic material [19] (Beryl Material) and isotropic material [20] (Brass, $\sigma = 0.33$). The values of angular speed required for initial yielding Ω_i^2 and fully plastic state Ω_f^2 has been given in Table 2. From table 2, it is observed that thin rotating disc made of isotropic material (Brass) having variable density m=-1 (density increases radially) with no edge load (σ_0=0) yields at the bore at a higher angular speed as compare to disc made of transversely isotropic material (Beryl), but with edge load, the rotating disc made of transversely/isotropic material yields at the bore at a lesser angular speed and at a much lesser angular speed with further increase in load. For m=1 (density decreases radially from the internal surface of the disc to the outer surface), it has been seen from the table 2 that rotating disc made of transversely isotropic material/ isotropic material with edge load yields at the bore at a much less angular speed as compare to rotating disc having variable density m=-1(density increases radially).

Rotating disc made of isotropic material having variable density with no edge load (σ_0=0) become fully-plastic at a higher percentage increase in angular speed from initial yielding as compare to rotating disc made of transversely isotropic material having variable density. With edge load, it requires much higher percentage increase in angular speed to become fully-plastic. It means that a thin rotating disc made of isotropic material whose density increases radially requires higher percentage increase in angular speed to become fully-plastic as compare to rotating disc having constant density or whose density decreases radially and made of transversely isotropic material. This percentage in angular speed further increases with the increase in edge load σ_0.

In Figures 2, 3, 4; curves have been drawn between stresses for a thin rotating disc having variable density m=-1,0,1 with edge load σ_0=0.0, 0.2, 0.3 and radii ratio R=r/b. It has been observed that circumferential stress is maximum at the bore for rotating disc made of isotropic material with no edge load (σ_0=0) as compare to rotating disc made of transversely isotropic material. The values of circumferential stress at the bore further increase with edge load. It means that rotating disc having variable density and made of isotropic material have a tendency to fracture at the bore i.e., it is where the largest tensile stress occurs as compare to rotating disc made of transversely isotropic material. Similar results were also shown by Rimrott [21] for plastic behaviour of rotating isotropic cylinder. The tendency of fracture at the bore increases with the increase in edge load. It is also observed from Figures 5, 6 and 7 for the rotating disc having variable density and edge load has no effect on the circumferential stresses for fully-plastic state.

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Table 1. Elastic constants C_{ij} (In units of 10^{10} N/m^2)

	C_{44}	C_{11}	C_{12}	C_{13}
Transversely Isotropic Material (C_2 = 0.69**, Beryl**)	0.883	2.746	0.980	0.674
Isotropic Material ($\sigma = 0.33$/C=0.50, **Brass**)	0.99997	3.0	1.0	1.0

Table 2. The values of angular speed required for initial yielding (Ω_i^2) and fully plastic state (Ω_f^2)

<table>
<tr><th rowspan="3">Density</th><th rowspan="3">Angular Speed</th><th colspan="6">Transversely Isotropic Material ($C_2 = 0.69$, Beryl)</th><th colspan="6">Isotropic Material ($\sigma = 0.33$, Brass)</th></tr>
<tr><th rowspan="2">$\sigma_a = 0$</th><th rowspan="2">$\sigma_a = .2$</th><th rowspan="2">$\sigma_a = .3$</th><th colspan="3">$P = \left[\sqrt{\frac{\Omega_f^2}{\Omega_i^2}} - 1\right] \times 100$ is %age increase in angular speed from initial yielding to fully –plastic state for different edge loading.</th><th rowspan="2">$\sigma_a = 0$</th><th rowspan="2">$\sigma_a = .2$</th><th rowspan="2">$\sigma_a = 0.3$</th><th colspan="3">$P = \left[\sqrt{\frac{\Omega_f^2}{\Omega_i^2}} - 1\right] \times 100$ is %age increase in angular speed from initial yielding to fully –plastic state for different edge loading.</th></tr>
<tr><th>$\sigma_a = 0$</th><th>$\sigma_a = .2$</th><th>$\sigma_a = .3$</th><th>$\sigma_a = 0.0$</th><th>$\sigma_a = 0.2$</th><th>$\sigma_a = 0.3$</th></tr>
<tr><td rowspan="2">m= -1</td><td>Ω_i^2</td><td>1.64</td><td>0.79</td><td>0.36</td><td rowspan="2">13.964</td><td rowspan="2">27.289</td><td rowspan="2">53.659</td><td>1.66</td><td>0.8102</td><td>0.38</td><td rowspan="2">22.720</td><td rowspan="2">42.537</td><td rowspan="2">78.443</td></tr>
<tr><td>Ω_f^2</td><td>2.13</td><td>1.28</td><td>0.85</td><td>2.50</td><td>1.646</td><td>1.21</td></tr>
<tr><td rowspan="2">m= 0</td><td>Ω_i^2</td><td>1.32</td><td>0.63</td><td>0.29</td><td rowspan="2">13.818</td><td rowspan="2">27.241</td><td rowspan="2">53.128</td><td>1.33</td><td>0.651</td><td>0.308</td><td rowspan="2">22.627</td><td rowspan="2">42.395</td><td rowspan="2">78.358</td></tr>
<tr><td>Ω_f^2</td><td>1.71</td><td>1.02</td><td>0.68</td><td>2.0</td><td>1.32</td><td>0.9798</td></tr>
<tr><td rowspan="2">m= 1</td><td>Ω_i^2</td><td>1.03</td><td>0.50</td><td>0.231</td><td rowspan="2">13.633</td><td rowspan="2">26.491</td><td rowspan="2">51.471</td><td>1.039</td><td>0.506</td><td>0.2496</td><td rowspan="2">22.533</td><td rowspan="2">41.979</td><td rowspan="2">78.304</td></tr>
<tr><td>Ω_f^2</td><td>1.33</td><td>0.8</td><td>0.53</td><td>1.56</td><td>1.02</td><td>0.76</td></tr>
</table>

Figure 1. Elastic-plastic Transitional Stresses in a Thin Rotating Disc for Different Edge Load (σ_o) and variable Density (m=-1)

Figure 2. Elastic-plastic Transitional Stresses in a Thin Rotating Disc for Different Edge Load (σ_o) and variable Density (m=0)

Figure 3. Elastic-plastic Transitional Stresses in a Thin Rotating Disc for Different Edge Load (σ_o) and variable Density (m=1)

Figure 4. Fully-plastic Stresses in a Thin Rotating Disc for Different Edge Load (σ_o) and variable Density (m=-1)

Figure 5. Fully-plastic Stresses in a Thin Rotating Disc for Different Edge Load (σ_o) and variable Density (m=0)

Figure 6. Fully-plastic Stresses in a Thin Rotating Disc for Different Edge Load (σ_o) and variable Density (m=1)